

## **Guaranteed Generation from Large Language Models**

Minbeom Klm, Thibaut Thonet, Jos Rozen, Hwaran Lee, Kyomin Jung, Marc Dymetman









LLMs will be increasingly adopted in critical decision-making.

The more important decisions an LLM has to make, even a slight leakage of undesired attributes can have severe consequences.

## We want 20% strict guarantees on our requirements!!!

Three Goals of Guaranteed Generation

- 1. How can we guarantee that all generated sequences from LLMs meet specific constraints or requirements?
- 2. How can we achieve this while preserving the original model's useful distribution as much as possible?
- 3. How can we simultaneously obtain the two previous properties at run-time with a limited inference cost?

- Base LLM: a(y)
  - Sampling:  $y \sim a(y)$
- The guarantee: any type of hard constraint b(y)
  - $b(y) \in \{0, 1\}$
  - b(y) = 1: guarantee satisfaction
  - b(y) = 0: guarantee violation
- The guarantee can be a simple or complex predicate
  - E.g. b(y) is a toxicity detector (0 means toxic, 1 means non-toxic)
  - E.g. b(y) is a verifier of certain keywords (0 means 'does not contain keyword', 1 means 'contains keyword')
  - E.g. b(y) also can be complex checker: Personal information leakage to unauthorized people
- Guaranteed sampler: sampler that **never** violates the b(y) guarantee.

We want guaranteed sampler that distorts a(y) as little as possible while conditioning on b(y), we define the target model as

$$g(y) \propto a(y)b(y)$$

# filtered model

$$g(y) = \frac{1}{Z}a(y)b(y),$$
$$Z = \sum_{y} a(y)b(y)$$

# target model

$$Z = \sum_{y} a(y)b(y)$$

# normalizer

- g(y) = a(y | b(y) = 1)
- Our target model g minimally deviates from a to satisfy the constraint b, which can be measured by  $KL(g \mid a)$ .
- That is why we target g(y) !!!

- We want to obtain **exact sampler:**  $y \sim g(y)$ . Can we obtain it with autoregressive models?
- First, it is impossible in general to find autoregressive model a'(y) = g(y),  $(P \neq NP)$  [1]
- Second, fine-tuning cannot produce an exact sampler.

A fundamental impossibility result We define a function from  $V^*$  to  $\mathbb{R}^d$  as polynomial-time computable (PTC) if and only if it can be computed in polynomial time relative to the length of its argument. A binary predicate is said to be PTC if it is a PTC function of its argument. An ARM a is considered PTC if and only if the computation of the softmax vector at each step t is a PTC function of the prefix up to that step.

The following result was inspired by the pioneering work of Lin et al. (2020b) on the general limitations of ARMs. See App. C.3 for a detailed statement and self-contained proof, adapted to the case of filtered models.

**Theorem 1.** Under the assumption  $P \neq NP$ , there exists a PTC ARM a and a PTC binary predicate b such that no PTC ARM a' has the same distribution as g.

[1] Limitations of Autoregressive Models and Their Alternatives, C.Lin et al.

To obtain Ideal distribution 
$$g(y) = \begin{cases} 0, & b(y) = 0 \\ \frac{1}{Z_{a,b}} a(y), & b(y) = 1, \end{cases}$$

Rejection sampling is an exact sampler!

1) 
$$y \sim a(y)$$
  
2) If  $b(y) = 1$ , return  $y$ ; else, go to 1

- Acceptance rate  $(AR_a)$  of this system is  $AR_a = \sum_y a(y)b(y)$
- If  $AR_a$  is high enough to directly use, then perfect!
- Unfortunately,  $AR_a$  is low for various general use cases = heavy inference costs.
- We need a more efficient approach!

- We target guaranteed samplers under b(y) and also almost exact sampler to g(y)
- Option 1: Construct faster system with autoregressive model a'(y), but give up on the guarantees
- Option 2: Approximate g(y) by MCMC sampling techniques, but it also require heavy inference costs.
- Option 3: Rejection sampling a'(y) on proxy target  $g'(y) \propto a'(y)b(y)$

1) 
$$y \sim a'(y)$$
  
2) If  $b(y) = 1$ , return  $y$ ; else, go to 1

- Yes, it is guaranteed and exact sampler of g'(y), but not of g(y).
- So, critical problem is the 'quality' of g'(y) toward g(y).
- If  $KL(g \mid g')$  is too high, we could try to improve a'(y) so that this divergence becomes smaller.
- New Goals with proposal a': High quality of  $KL(g \mid g')$  with good efficiency  $AR_{a'}$

• GUARD's key theorem for optimizing proposal distribution a'(y)

Theorem 2: 
$$KL(g \mid a') = -\log AR'_a + KL(g \mid g')$$
, objective efficiency closeness

• We prove that  $KL(g \mid a')$  minimization jointly optimize better efficiency and distributional closeness to our sampling target.

• Conclusion: Let's minimize  $KL(g \mid a')$  for guaranteed and almost exact sampler to target distribution g!!!

- 1. Constraint-aware Prompting (CAP): Give prompts to encourage the output y to respect the given constraint b(y) = 1
- +: Do not need any sample from g.
- -: There is no guarantee to minimize  $KL(g \mid a') = KL(g \mid a, CAP)$

- 2. Supevised Fine-Tuning (SFT)
  - 1) Sample a large number of  $y \sim a$ , then filter them only b(y) = 1 to create dataset representing g.
  - 2) Then, fine-tune a with this dataset to train proposal  $a'_{\theta}$ .
- +: Theoretically guaranteed to minimize  $KL(g \mid a'_{\theta})$
- -: We need to sample a lot of  $y \sim a$  to create a sufficient dataset.

- 3. Distributional Policy Gradient (DPG): with proposal  $a'_{\theta}$ ,
  - 1) Sample  $y \sim a'_{\theta}$  with adaptive proposal, then filter them only b(y) = 1.
  - 2) Then, train  $a'_{\theta}$  with DPG loss  $\nabla_{\theta} KL(g \mid a'_{\theta}) = -\mathbb{E}_{y \sim g} \nabla_{\theta} \log \pi_{\theta}(y) = -\mathbb{E}_{y \sim \pi_{\theta}} \frac{g(y)}{\pi_{\theta}(y)} \nabla_{\theta} \log \pi_{\theta}(y)$
- +: Theoretically guaranteed to minimize  $KL(g \mid a'_{\theta})$
- +: As the proposal  $a'_{\theta}$  approaches g, the frequency and quality of gradient updates increase, making it more efficient.
- -: Still inefficient in the early stage when  $a_{\theta}' \simeq a$ .
- 4. Warm-start DPG (adopted as GUARD Training algorithm)
  - 1) Sample  $y \sim a(\cdot | CAP)$ , then filter them only b(y) = 1 and fine-tune a with this dataset to make initial point  $a'_{\theta}$ .
  - 2) Start DPG training on wart-starting point.
- +: Theoretically guaranteed to minimize  $KL(g \mid a'_{\theta})$
- +: As the proposal  $a'_{\theta}$  approaches g, the frequency and quality of gradient updates increase, making it more efficient.
- +: Skip the slow early exploration stage where training samples are scarce.

Unconditional text generation under lexical constraints

Model a: Gemma-2B

Goals: Starting with the  $\langle bos \rangle$  token, a generates a text y of 30 tokens with keywords "amazing"

Binary constraint b: b(y) = 1 iff y contains the string "amazing"

$$AR_a$$
 of  $b \sim 0.0023$ 

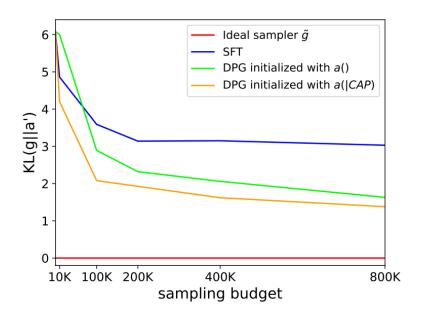
Conditional text generation under sentiment reversal

Model a: GPT-2 finetuned on ROCStories Sentiment scorer  $b_{score}$ :  $\bigcirc$  michellejieli/emotion\_text\_classifier

Goals: Start with very negative story opening X ( $b_{score}(X) < 0.05$ ), complete positive ending story y = [Y; Z].

Binary constraint b: b(y) = 1 iff the final sentence Z is very positive  $b_{score}(Z) > 0.98$ .

$$AR_a$$
 of  $b \sim 0.005$ 



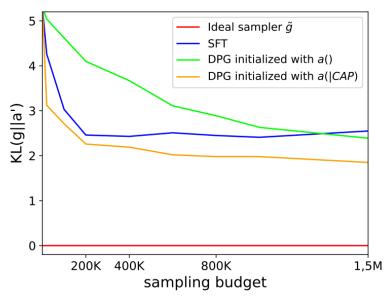
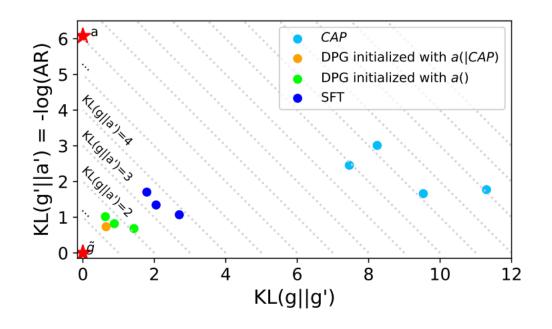
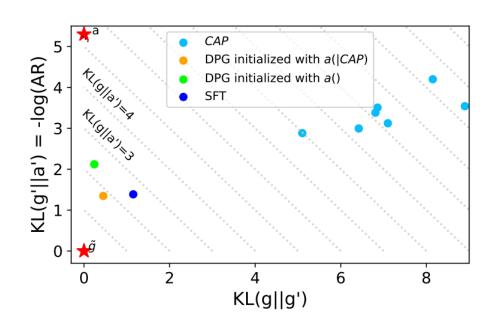


Figure 3: Evolution of KL(g||a') as a function of the number of samples used for training with lexical constraints.

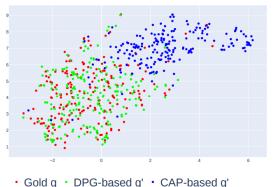
Figure 5: Evolution of KL(g||a') as a function of the number of samples used for training in sentiment reversal.

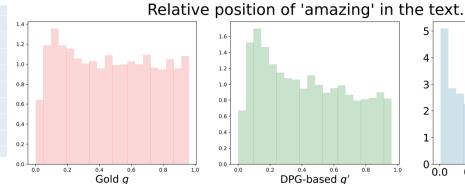
- With adaptive proposal a', DPG shows better optimization than SFT.
- Warm-start DPG avoid inefficient early stage with off-policy sampler  $a(\cdot \mid CAP)$ .

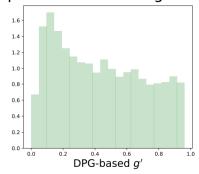


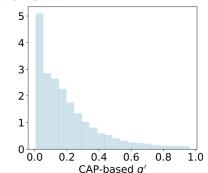


- CAP achieve nice  $AR_{a'}$ , but really diverged in terms of  $KL(g \mid g')$ .
- Within the same sampling budget, warm-start DPG shows the best Pareto-optimality.
- While preserving a high proximity to g, AR improvement is 180x (0.0023 -> 0.416) and 60x (0.005 -> 0.306)



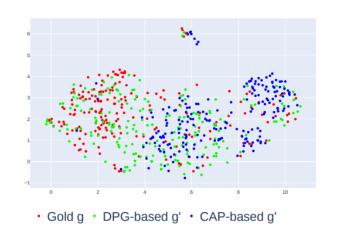


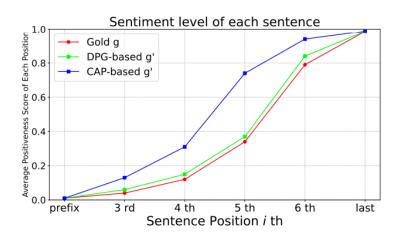




Approach	Self-BLEU ↓				Semantic
	2-gram	3-gram	4-gram	5-gram	similarity ↓
Gold g	0.315	0.117	0.057	0.032	0.365
DPG-based $g'$	0.342	0.124	0.067	0.040	0.372
CAP-based $g'$	0.438	0.260	0.193	0.156	0.488

- Also in our analysis, we can detect distributional distortion with CAP: positional bias and loss of diversity.
- But, GUARD training preserves these properties as much as possible.
- GUARD naturally satisfies the "keywords" constraints without overconsciousness.





Approach		Semantic			
	2-gram	3-gram	4-gram	5-gram	similarity ↓
Gold g	0.744	0.569	0.458	0.337	0.876
DPG-based $g'$	0.753	0.578	0.469	0.344	0.868
CAP-based $g'$	0.827	0.701	0.621	0.554	0.956

- Also in our analysis, we can detect distributional distortion with CAP: positional bias and loss of diversity.
- But, GUARD training preserves these properties as much as possible.
- GUARD naturally satisfies the "positive ending" constraints without overconsciousness.

- We hope **guarantees** in LLMs while **preserving original distribution** as much as possible.
- First, we formalize what is the optimal distribution g for guaranteed generation.
- And this distribution g cannot achieved only with training, that is why we need harmonization between trainingtime & inference-time methods.
- We present a **theorem** in the form of an equality that **jointly optimizes** distributional closeness and acceptance rate, while rejection sampling fixed in inference-time. And **suggests GUARD baselines: CAP, SFT, and DPG**.
- Experimentally, we confirmed that warm-start DPG maintains proximity to the ideal distribution while increasing the acceptance rate by nearly a 100x, and various analyses support this finding.

- Two types of tasks: Relativity and Absoluteness
- Human preference is **relative**, but reasoning, safety, code compilability is **absolutely correct or wrong**.
- But, extremely many previous works do RL style training for absolute tasks.

KL-regularized RL's ideal distribution:  $g(y|x) \propto a(y|x) \exp(\frac{b(x)}{\beta})$ 

- -> Allocates non-zero probability for absolutely wrong output y.
- $\rightarrow \beta$  decides how trained model can be diverged from a.

**GUARD**'s ideal distribution:  $g(y|x) \propto a(y|x)b(x)$ 

- -> Allocates **zero probability** for absolutely wrong output y.
- -> It is the closest distribution from a with perfectly satisfying constraint b(y).

It is why we should pay attention to GUARD training objective for solving absolute tasks.

## Thank you!